APPENDIX A
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"(A) NON-CONVENTIONAL COMMUNICATION DEVICES FOR THE MARS MISSION, AND (B) USE OF RADAR TECHNIQUES TO SELECT MARS LANDING SITE"

by

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(A) NON-CONVENTIONAL COMMUNICATION DEVICES FOR THE MARS MISSION*

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The advantages which are obtained from the use of non-conventional communication systems operating at wavelengths other than microwave, e.g., optical and millimeter, are weighed against the disadvantages. It is concluded that some small relative advantage can probably be gained in the millimeter range, but that this advantage is not significant and that there is no foreseeable advantage in the optical range. Therefore, it appears that improvements in space communication techniques are most likely to be found within the microwave region.

Let me begin by making a small but significant grammatical addition to the title of the first part of my talk; I want to add a question mark to the end of the title. As Dr. Rechtin has indicated, adequate if not all-satisfying communication for the manned Mars exploration can be expected from currently used devices and techniques, foreseeably improved over the next few years. Hence, before the accumulated experience with these techniques is abandoned, a rather strong case must be made for a change.

Of the newer devices often referred to in connection with space communication, the laser is often described as particularly promising. I am inclined to think that even granting considerable improvement over current laser performance, it is not at all clear that the laser is overly promising for this application.

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*Prepared for technical papers that will later be published in the Proceedings of the American Astronautical Society.

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Suppose we consider an optical communication system relative to a microwave system. I will talk only about communications from Mars to Earth, the difficult link. Further, I'm not going to worry about atmospheric interference problems; I'll put the receiver outside the Earth's atmosphere, in a satellite or on the Moon, and the transmitter outside Mars' atmosphere.

As a measure of the relative merit of the two systems, suppose we use a signal-to-noise ratio - the ratio of the power available to a detector, to noise power. Leaving aside miscellaneous constants the factors involved in the comparison are:

1. Power radiated by the antenna, P_T;
2. Gain of the transmitting antenna, for which I'll use a measure of beamwidth, θ;
3. Effective area of the receiving antenna, A_R;
4. Noise power, for which I use the product of an effective temperature and bandwidth, $T\Delta f$;
5. Detection efficiency, the efficiency of conversion of power incident on the receiving antenna to power available to the receiver input amplifier, E_R.

Thus:

$$\underset{\text{Signal-to-Noise Power at Detector}}{\frac{S}{N}} \approx \underset{\text{Radiated Power}}{[P_T]} \times \underset{\text{Transmitter Beamwidth}}{[\theta^2]} \times \underset{\text{Area of Receiving Antenna}}{[A_R]} \times \underset{\text{Detection Efficiency}}{[E_R]} \times \left[\frac{1}{\text{Effective Temperature X Bandwidth}}\right].$$

We might make a small change in interpretation. If a pulse coded signal is assumed, the bandwidth Δf can be considered to be a reciprocal pulse period, and $P_T/\Delta f$ considered as the energy per pulse - or, crudely, as the energy per bit of information for low S/N ratios.

Let us consider some relative values for these parameters.

First we might note that radiated power P_T is probably not the relevant parameter for the comparison. A more pertinent parameter is the efficiency of conversion from prime energy to radiated energy. The available prime energy will be determined primarily by the vehicular considerations. The fraction of it allocated for communication should be a small fraction of that devoted for continuous operation of the life support system, or possibly an ion propulsion system. Even this small fraction is further reduced by the need for a certain amount of redundancy which will be used for reliability assurance.

Current microwave systems have conversion efficiencies ranging from 10 to 40 percent; we might expect to have available 50 percent conversion efficiency. Since these systems have been around quite awhile, it is not likely that major improvements will be made.

Solid-state and gas lasers have efficiencies running to a few percent. Infrared diode lasers have efficiencies comparable to microwave systems, although with considerably less average power capability than solid-state lasers.

Nevertheless, I will allow the laser marked improvement, and assume that both systems will have comparable efficiencies. In effect, then I'll be comparing the two systems with both having the same radiated power.

The transmitter beamwidth θ is the next factor; this is the factor which makes the optical system <u>seem</u> attractive. Roughly speaking, the limiting beamwidth is of the order of the ratio of wavelength to a characteristic antenna dimension. If we suppose the microwave wavelength to be 10 centimeters (S-band) and the optical wavelength to be 1 micron (Ruby at .6943 microns, or gas at 1.15μ), it would appear that for the same antenna dimensions we could get a factor of 10^5 improvement in beamwidth, and thus much more effectively focus the transmitted power onto the receiving antenna. In principle, this is quite true. In practice, this resolution would be a little hard to use. At a nominal range of 40×10^6 miles, a beamwidth of 10^{-5} radians focusses to a "spot" diameter of only 400 miles. A pointing error or pointing fluctuation of 10^{-5} radians moves the beam off the receiving antenna. And, to achieve this kind of focussing requires maintaining antenna tolerances to, say, $\lambda/20 = 5$ Å or better. I don't want to be unduly pessimistic about the optical beamwidth, but I think that a number like 10^{-4} radians is more realistic than the diffraction limit of 10^{-6} radians which could, in principle, be achieved with about a three-foot mirror.

As far as the microwave antenna is concerned, it seems reasonable to suppose that a rather large antenna could be carried to Mars disassembled, and re-assembled within quite reasonable dimensional tolerances at Mars, particularly so if assembly is made in orbit, where structural strength and rigidity requirements are simplified. Being a little optimistic I will assume that a microwave beamwidth of 10^{-2} radians is a reasonable expectation.

Squaring the beamwidths, and taking the ratio, I get a factor of 10^{4} in favor of the optical system at the transmitter end.

Now let's turn to the area of the receiving antenna. Some idea of the maximum optical aperture which can be anticipated is obtained by referring to the 200-inch telescope at Mount Palomar. To be fair, we should contrast this with, say, a 250-foot microwave antenna. I don't want to belabor the question of construction problems, dimensional tolerances and so forth. Nor do I think it necessary to dwell on possibilities of building up a large effective area by stacking individual small antennas. For this comparison, I think it is reasonable to give the microwave system an advantage of a nominal 100 to 1 area ratio for coherent reception.

When it comes to noise power, specifically the effective noise temperature, the optical system operates at a disadvantage. The quantity corresponding to the spectral power density kT at microwave frequencies is replaced at optical frequencies by hf. A corresponding equivalent temperature is T equivalent = hf/k and, at 1 μ, is about 10^{4} degrees. For the microwave receiver I will use a nominal 100 degrees, and thus favor the microwave system by a factor of 10^{2}.

For a microwave system a nominal 50 percent of the power collected by the antenna is transferred to the receiver input. Quantum conversion efficiencies at optical frequencies are, in general, considerably lower. Photoemissive surfaces have photon-to-electron power conversion efficiencies of a few percent, lower in the red-infrared part of the spectrum. Photo conductros and photo junction devices approach unity quantum efficiencies, but they are in general relatively high signal level devices. I'll be generous to optics again and call the relative weighting factor 1.

If all the transmitter and receiver weighting factors are multiplied, the overall weighting factor is <u>unity</u>, i.e., for the same radiated power both systems have the same signal-to-noise ratio.

The fact that it is unity is not so important; numbers can be juggled one way or the other. What is important is that it is not an obviously large factor even with the assumption of a considerable improvement in laser performance. If we keep in mind the reasonable presumption that the energy allocated to communications will be a small fraction of the total available energy, it is a moot point whether any small savings in weight and size which might accrue to the laser system would be worth the sacrifice of the accumulated understanding, experience, and availability of conventional microwave equipments.

The conclusion I have come to regarding detected signal-to-noise ratio is, roughly speaking, not critically dependent on the choice of wavelength. This then suggests an examination of the spectrum between, say, ten centimeters and one micron for a wavelength which combines the desirable aspects of both wavelengths to best advantage.

Generally speaking, we would like a wavelength for which:

1. An antenna of a few meters dimension has a beamwidth near the best useable,
2. The noise spectral power density is described more by kT than by hf,
3. A fair amount of understanding, experience, and equipment exists for generating, amplifying, modulating, and detecting signal exists,
4. Structural and dimensional tolerances are not overstringent.

I think it is clear that I am talking in terms of millimeter wavelengths. Capability exists at present of generating about a watt of cw power at one millimeter. An antenna diameter of about five meters will produce a beamwidth near to 10^{-4} radians. It is not infeasible to consider a medium temperature (300°K) maser preamplifier for the receiver, using super conducting magnets to generate the needed magnetic field. Using such parameters, one can show that the millimeter waves offer some slight advantages; but the margin hardly outweighs the reliability attainable using established conventional microwave techniques. However, it does seem to me that the millimeter waves offer at least as good, and probably a better, likelihood than optical wavelengths of competing with conventional techniques.

There is one other factor in the range equation I used earlier that I did not mention explicitly, and that is range. A factor of about four might be cut from the power or beamwidth requirements for a single transmitter by putting up a relay transmitter halfway between Mars and Earth. This reward is, however, insignificant compared to the problems of maintaining the relay in the right orbit, at the right time, with the right orientation.

Last, I would like to acknowledge the assistance given me in assimilating the data for this brief review by Dr. Murray Miller of our staff.

(B) THE USE OF RADAR TECHNIQUES
TO SELECT A MARTIAN LANDING SITE*

Keeve M. Siegel+

A set of experiments to determine the geometrical character and material composition of the Martian surface are described. These experiments depend upon both passive and active electromagnetic radiation measurements. Bistatic and monostatic, multi-wavelength, high resolution radar mapping experiments are strongly recommended.

The selection of a landing site in a human exploration of Mars should be based on two types of information. One is the shape of the surface contour: the size and shape of protuberances which may interfere with a safe landing and lift-off. The other is the physical nature of the surface material, particularly its structural properties.

The most reliable information about the surface contour, in the detail which would be required, will presumably be obtained by photographic means, provided that conditions for propagation in the Martian atmosphere will permit this. Detailed topographical maps will be necessary for making a judgment about the structural stability of a landing site, and pictures having a resolution in the neighborhood of a foot or so and taken at various aspect angles would be necessary to satisfy the requirement completely. The collection, interpretation, and communication of images of this scale will pose serious mission time problems.

The inherently more difficult question concerning the material composition of the landing surface can be answered without surface contact only through the use of several correlated experiments which involve radiation at a number of

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frequencies. By means of such experiments based upon the observation of electromagnetic radiation at various frequencies, the desired structural information can be obtained indirectly, in spite of the fact that the actual measurements will be dominated by material properties other than structural strength.

Two fundamental types of observation of electromagnetic radiation at radio frequencies are potentially useful for this purpose. One is a radiometric measurement of the passive radiation from the surface. This type of measurement is at present a standard item in space exploration programs. The other is an active radar scattering measurement. Both types of observation produce data which can be used to characterize material properties from the outer surface to some depth below it, thus providing information which could not be obtained by photographic observation alone.

Measurements of passive radiation at various frequencies will lead to estimates of the product of the thermal conductivity and volumetric specific heat of the surface material. Estimates of other thermal constants can be deduced from radiometric measurements of the surface radio temperature distribution as a function of time and position on Mars relative to the sun. Such estimates have been made, for example, from observations of the thermal emission from the Earth's moon by several investigators, cf. Giraud[1], who has summarized most of the work which has been done in this area and has presented an extensive bibliography of it. Results obtained from observations of this nature can be helpful in determining the material composition of the Martian surface and in checking other mathematically related material constants which may be determined from data produced by independent experiments of a different type.

In particular, one can use the fact that the instantaneous surface temperature at a given surface element is proportional to the power reflection coefficient as a function of the frequency and the angle of the radiation.[2] The proportionality factor depends upon thermal properties of the material in a given area as well as the amount and duration of the exposure of that area to the sun. Relations between various physical parameters involved have been discussed by Giraud in connection with the application to the Lunar surface measurements. Of course, some allowance should be made for the effect of the Martian atmosphere in the present case. The detailed information which can be obtained as well as the reliability of the method depends most crucially upon the resolution capability of the radiometer used.

A second type of experiment utilizes an active radar, and it can be effectively employed in conjunction with passive radiation experiments to determine the material composition of the Martian surface. A simple measurement of the active type, for example, can be used to estimate the degree of smoothness of the surface. This can be done by means of a monostatic radar measurement at a given wavelength which then sets the scale of the roughness whose presence or absence will be determined by that particular observation. The measurement is implemented by the transmission of a linearly polarized wave and the reception of both the parallel and orthogonal components of the return. The degree of smoothness relative to the wavelength used will be determined from an observation of the ratio of the amplitudes which correspond to the main bang return of the two received orthogonal signals. A rough surface, that is, one having many protuberances whose dimensions are nearly of the same order of magnitude as the wavelength, will scatter equal amounts of energy polarized in both directions, whereas a smooth surface will tend to scatter most of the energy in the form of waves polarized in the same direction as that of the incident wave and very little of the energy returned will be in the form of waves polarized in the orthogonal direction.

The electromagnetic constants of the outer surface layer, and possibly of layers underneath, depending on the nature of the surface material, can be determined along with the depth of the outer layer by radar measurements on a particular area. A method for doing this depends upon the utilization of a number of different frequencies. An alternate procedure, however, can be based upon the use of frequency scanning at a sufficiently rapid rate instead of the simultaneous measurements at different frequencies. A serial-parallel sampling technique can be employed in this connection.

The experiment will yield depth information only if the imaginary part of the surface dielectric constant is small at the frequencies employed. Also, in certain areas where exponential gradients exist in the outer layer or where the outer layer has strong magnetic properties depth measurements which depend on the use of this technique are not possible. It can also happen that some geometrical configurations of the inner layer may lead to ambiguous depth measurements; however, a considerable amount of significant information should still be obtainable even in this case.

To illustrate how data obtained from direct radar measurements can be utilized, let us, for the moment, assume that there are two surface layers of which the inner one is dense and the outer one less dense and of thickness d. If the outer shell has an index of refraction n and the inner shell an index m, the voltage reflection coefficient R may be represented in the form

$$R = R_1 + R_2 \, e^{2j\beta d} \quad ;$$

β is the propagation constant in the material and is equal to $2\pi/\lambda_n$, where λ_n is the wavelength in the outer layer; R_1 and R_2 are the voltage reflection coefficients of the outer and inner layers, respectively, and may be computed from

$$R_1 = \frac{1 - n}{1 + n} \, , \qquad R_2 = \frac{1 - \frac{m}{n}}{1 + \frac{m}{n}} \, .$$

The indices of refraction are given by

$$n = \sqrt{\frac{\mu_o \epsilon_1}{\mu_1 \epsilon_o} \left(1 + j \frac{s_1}{\omega\epsilon_1}\right)} \, ,$$

$$m = \sqrt{\frac{\mu_o \epsilon_2}{\mu_2 \epsilon_o} \left(1 + j \frac{s_2}{\omega\epsilon_2}\right)} \, ,$$

where μ is permeability, ϵ is permittivity, s is conductivity, and the subscripts 0, 1, 2 refer, respectively, to free space, the outer layer and the inner layer.

To simplify the discussion, let us assume, first, that the conductivity s of the inner and outer layers is zero. From this assumption it follows that R_1 and R_2 are real quantities. The equation for R now describes a simple vector sum with the vector $R_2\underline{/2\beta d}$ rotating about the vector $R_1\underline{/0}$ as a function of the wavelength and depth. From this it is seen that R will be a minimum whenever the thickness of the outer layer is an odd number of quarter wavelengths. A maximum will occur whenever the thickness is an integral number of half wavelengths. It therefore seems reasonable to radiate, say, the fundamental and second harmonic frequencies simultaneously or alternately and to look for resonances in the received data.

Of course, it is not likely that the conductivity is zero as assumed. A finite conductivity will cause the signal penetrating the outer layer to be attenuated, and, therefore, as the wavelength becomes small compared to the depth the resonances will be damped and more difficult to detect. Also, if the material has conductivity the resonance points will be shifted from the multiple quarter wavelength intervals. Generally, a well-defined minimum will occur when the depth of the outer layer is slightly less than a quarter wavelength in this case.

Thus, the radar scattering experiment to determine depth should be conducted at fairly long wavelengths, and a range of frequencies should be used. One could also perform the experiment in such a way as to determine the reflection coefficients of the surface material. Since the reflection coefficient depends on the quantities ϵ_1/μ_1, s_1/μ_1, ϵ_2/μ_2, s_2/μ_2, and $d\mu$, it is clear that simultaneous measurements with at least five different frequencies should be made to determine the five unknowns.

In addition to this kind of measurement, aimed at obtaining information about the surface material and a rough estimate of the depth of the outer surface layer, a more sophisticated radar experiment is also possible. The synthetic antenna principle in connection with a side-looking radar can be utilized to obtain a high resolution radar map of the Martian surface, and, in fact, of the inner surface layer if the appropriate frequency needed for penetration to the required depth is used. This frequency will be known from the data obtained in the previous radar experiment. The synthetic antenna principle will permit the construction of a contour map of the inner surface having fine resolution in both the range and azimuth directions.

To implement the high-resolution mapping experiment, an altimeter to determine the height of the vehicle will probably be needed. In addition, the vehicle velocity relative to the surface should be controlled or at least measured since this knowledge would be required for processing the radar data in order to construct the final map.

Resolution of about one-half mile can be obtained from bistatic measurements with the transmitter in the orbitor or fly-by vehicle and the receiver on the ground. For finer resolution the transmitter will be on the ground where more weight can be added without penalty. The receiver would be in the orbitor or fly-by vehicle with storage capability to allow for a lower data rate for telemetry transmission to the ground in order to reduce bandwidth and power requirements in the orbitor or in the fly-by vehicle.

The successful implementation of all of the above experiments will provide a considerable amount of information pertaining to the geometrical and physical nature of the Martian surface and its substructure. This information should be of inestimable value in the selection of a safe landing site. Therefore, it would clearly be worth a great deal of technological effort to provide a system capable of supporting such an experimental program.

I would like to express my appreciation to Dr. Irvin Kay of our staff for his contributions in the preparation of this paper.

REFERENCES

1. A. Giraud, "Characteristics of the Moon's Surface Layer; An Analysis of its Radio Emission," Astrophy. J., 135 (1), 1962, pp. 175-186.
2. Cf. Giraud loc.cit. and V.S. Troitskii, "A Contribution of the Theory of Radio Emission of the Moon," Astr. Zhun., 31 (6), 1954, pp. 511-528.

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APPENDIX B

MEMO TO: Jack Miller May 6, 1963

FROM: David Galbraith

SUBJECT: A Calculation of Atmospheric Radiation Transfer

Introduction

The variety of methods used in calculating the effect of a given planetary atmosphere upon surface temperature appears to be in large measure responsible for the differences of opinion concerning the atmosphere of Venus. The majority of calculations on the Greenhouse effect appear to use an exponential relationship,

$$T_e^4 = e^{-\tau_s} T_s^4 \tag{1}$$

where T_e is the effective radiation temperature of the planet into space, T_s in the surface temperature, and τ_s the optical depth of the atmosphere for infrared radiation. Dr. Öpik (Ref. 4) used an expression which gives an energy flow decrease linear with optical depth,

$$T_e^4 = \frac{T_s^4}{1 + 1.5\tau_s} \tag{2}$$

This formula requires a much higher optical depth to produce the observed temperatures than did equation (1), and Öpik was led to conclude the Greenhouse effect would not of itself be capable of explaining Venus' temperatures. Probably the best formulation applied to the problem thus far is that of Ohring and Coté (Ref. 3), whose formula,

$$T_e^4 = \frac{T_s^4}{1/2\tau_s} \left\{\frac{1}{3} - E_4(\tau_s)\right\} \tag{3}$$

is exact for the case of no scattering. Ohring and Coté have shown that when an extensive, high cloud cover is taken into account, a sufficient Greenhouse effect may be maintained by a physically realizable atmosphere. Unfortunately, the mathematics of equation (3) requires an excessively large non-radiative transfer of heat from the atmosphere to the solid surface in order to maintain the heat balance. It thus appears that a more exact formulation of the atmospheric heat transfer problem is highly desirable at this time, to provide a basis for discussion of the various proposed models of the Cytherean atmosphere. The mathematics required for such a formulation is available, and is discussed in Chandrasekhar (Ref. 2) and Busbridge (Ref. 1). This formulation, which requires a numerical type of solution, is set forth below.

Derivation

Suppose we have an atmosphere which absorbs and scatters isotropically, and emits radiation in the range of interest isotropically according to an arbitrary function of position. If the mass density is ρ, the absorption coefficient k, the ratio of scattering to absorption is ω_o, and the emission function is $B'(x)$, the differential equation governing the intensity I of radiation in an arbitrary direction x is

$$\frac{\partial I}{\partial x} = k\rho\, I + k\rho \frac{\omega_o}{4\pi} \int I\, d\omega + \frac{k\rho}{\pi}\, B'(x) \qquad (4)$$

We shall assume that the only net variation with position is due to changes in the altitude Z, where $Z = X \cos\theta$. Thus the differential equation becomes

$$\frac{\cos\theta}{k\rho} \frac{\partial I}{\partial z} = -I(z,\theta) + \frac{\omega o}{2}\int_o^\pi I\,(z,\theta')\sin\theta' d\theta' + \frac{B''(z)}{\pi} \qquad (4a)$$

Finally, the representation is simplified somewhat by introducing the variables

$$\mu = \cos\theta$$

$$\tau = \int_z^{zo} k\rho dz'$$

where Z_o is an arbitrary, constant altitude (or infinity), and τ is known as the optical depth. Equation (4a) becomes

$$\mu\frac{\partial I}{\partial \tau} = I(\tau,\mu) - \frac{\omega_o}{2}\int_{-1}^{1} I\,(\tau,\mu')\, d\mu' - \frac{B(\tau)}{\pi} \qquad (5)$$

Equation (5) is strictly true only for a single frequency, but it is frequently acceptable to assume that the atmosphere is "grey" - i.e., the various parameters do not vary with frequency. In this case, equation (5) may be used to determine the total radiative intensity in the atmosphere.

Differential equation (5) is not exactly soluble except for the very simplest of physical models for an atmosphere. The method discussed below, however, is capable of providing solutions to any required degree of approximation, provided the source function $B(\tau)$ is known. For many planetary atmospheres, the lapse rate (temperature vs. altitude) may be assumed to be adiabatic, thus defining $B(\tau)$ for a given model of ρ and k as functions of altitude.

The solution to equation (5) is found in terms of the function

$$\mathcal{F}(\tau) = \frac{B(\tau)}{\pi} + \frac{\omega_o}{2} \int_{-1}^{1} I(\tau,\mu)d\mu \tag{6}$$

It is convenient to split up the radiation intensity $I(\tau,\mu)$ into two functions, $I(\tau,+\mu)$ and $I(\tau,-\mu)$, the outgoing (upward) and incoming intensities respectively. The parameter μ is then restricted to positive values only. With this notation, equation (5) becomes

$$\mu \frac{\partial I(\tau,+\mu)}{\partial\tau} = I(\tau,+\mu) - \frac{\omega_o}{2} \int_0^1 \{I(\tau,+\mu') + I(\tau,-\mu')\}d\mu' - \frac{B(\tau)}{\pi} \tag{5a}$$

$$\mu \frac{\partial I(\tau,-\mu)}{\partial\tau} = -I(\tau,-\mu) + \frac{\omega_o}{2} \int_0^1 \{I(\tau,+\mu') + I(\tau,-\mu')\}d\mu' + \frac{B(\tau)}{\pi} \tag{5b}$$

This splitting up is necessary because the boundary conditions are normally given as the incident radiation at either side of the atmosphere:

$$\begin{aligned} I_1(\mu) &= I(\tau_1, -\mu) \\ I_2(\mu) &= I(\tau_2, +\mu) \end{aligned} \tag{7}$$

where τ_1 is the optical depth at the upper boundary under consideration ($\tau_1 = 0$ if the whole atmosphere is considered), and τ_2 is the optical depth at the lower boundary.

We now define the function $\Psi(\tau)$ and the integral operator $L_\tau\{\Phi(t)\}$:

$$\Psi(\tau) = \frac{B(\tau)}{\pi} + \frac{\omega_o}{2}\int_0^1 I_1(\mu)\exp\{-\frac{(\tau-\tau_1)}{\mu}\}\,d\mu + \frac{\omega_o}{2}\int_0^1 I_2(\mu)\exp\{-\frac{(\tau_2-\tau)}{\mu}\}\,d\mu \tag{8}$$

$$L_\tau\{\Phi(t)\} = \frac{\omega_o}{2}\int_{\tau_1}^{\tau_2} \Phi(t)\,E_1(|t-\tau|)\,dt \tag{9}$$

where $E_1(x)$ is the $n = 1$ case of the exponential integral:

$$E_n(x) = \int_1^\infty \nu^{-n} e^{-x\nu}\,d\nu \tag{10}$$

It can be shown that with

$$\mathcal{F}(\tau) = \sum_{n=0}^{\infty} L_\tau^n\{\Psi(t)\} \tag{11}$$

the solutions to equations (5a) and (5b) are given by

$$I(\tau,+\mu) = I_2(\mu)\exp\{-\frac{(\tau_2-\tau)}{\mu}\} + \int_\tau^{\tau_2}\mathcal{F}(t)\exp\{-\frac{(t-\tau)}{\mu}\}\frac{dt}{\mu} \tag{12a}$$

$$I(\tau,-\mu) = I_1(\mu)\exp\{-\frac{(\tau-\tau_1)}{\mu}\} + \int_{\tau_1}^{\tau}\mathcal{F}(t)\exp\{-\frac{(\tau-t)}{\mu}\}\frac{dt}{\mu} \tag{12b}$$

In a numerical calculation, we obviously cannot find $\mathcal{F}(\tau)$ from equation (11) exactly. It can be shown that the function

$$\mathcal{F}_N(\tau) = \sum_{n=o}^{N} L_\tau^n\{\Psi(t)\} \tag{13}$$

converges to $\varphi(\tau)$ as $N \to \infty$. For ω_0 small, the convergence will be rapid, but as ω_0 approaches unity, N may have to be quite large to obtain a close approximation.

Discussion

There are several uses to which calculations based upon the foregoing formulation may be put. Of particular interest would be an extension of recent work by Ohring and Coté. As is mentioned in the introduction, their work requires a large non-radiative heat transfer from the atmosphere to the solid surface. It would appear likely, however, that calculations which take into account scattering as well as absorption in the atmosphere would show this heat transfer requirement to be smaller than is indicated by the purely absorptive calculations of Ohring and Coté. One would not expect this quantity to be eliminated entirely, but it might be reduced in size to something which could be accounted for by a process such as Öpik's wind friction. Since virtually nothing is known about Venus' atmosphere below the clouds, the calculations must be done for a wide variety of combinations of optical depth, scattering albedo ω_0, and percentage of cloud cover in order to establish feasible ranges of possibility for these parameters. Other calculations for which this formulation should be at least partially adaptable would include calculations of the height to which convective flow can be maintained in the atmosphere, both for the Greenhouse and for the Aeolosphere models.

Bibliography

1. Busbridge, I.W. "Mathematics of Radiative Transfer"
   Cambridge University Press 1960
2. Chandrasekhar, S. "Radiative Transfer"
   Dover 1960
3. Ohring, George and Coté, Owen "The Meteorology of Mars and Venus"
   Annual Technical Report, Contract No. NASw - 286
   Geophysics Corporation of America, January 1963
4. Öpik, E. J. "The Aeolosphere and Atmosphere of Venus"
   Journal of Geophysical Research 66 No. 9 2807-2819 (1961)